TAKING ACCOUNT OF RESONANCE RADIATION IN THE PROBLEM OF NONEQUILIBRIUM-IONIZED GAS FLOW AROUND BLUNT BODIES

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Hypersonic, nonequilibrium-ionized, monatomic, inviscid gas flow around blunt bodies is considered. The emergence of radiation in the heated layer from the ground layer in the continuum and in the resonant line is taken into account.

1. Formulation of the Problem for the Shock Layer

The hypersonic flow of a nonequilibrium-ionized, monotomic, inviscid, radiating gas flow around blunt bodies is investigated. Presented as an illustration is the computation of the flow of argon around bodies with a spherical bluntness. Collision ionization in the domain between the shock front and the body surface is taken into account in terms of the excited level and energy transmission from the heavy particles to the electron gas as a result of elastic collisions between electrons and ions and atoms, as well as in terms of photoionization from the ground and excited level and the transfer of resonance radiation.

The expression for the elastic and inelastic collision reaction rates as well as for the rate of photoionization from the ground level are presented in [1, 2, 3]. To take account of continuum radiation from the excited level in the shock layer, a volume deexcitation model was used [4].

Radiation transfer into the resonant line ($3p^6\,{}^1S_0 - 3p^5 4s\,{}^1p_1$) should especially be considered. Doppler, resonant, and Stark line broadening [5] were taken into account in computing the resonance absorption coefficient k_ν. Taking simultaneous account of these effects results in the following expression for k_ν [6]:

$$k_\nu = k_0 \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-y^2)\,dy}{a^2 + (\omega - y)^2}\,, \tag{1.1}$$

$$a = \frac{\Delta\lambda_r + \Delta\lambda_{1/2}}{\Delta\lambda_D} \sqrt{\ln 2}\,, \tag{1.2}$$

$$\omega = \frac{2\sqrt{\ln 2}\,\Delta\lambda}{\Delta\lambda_D}\,. \tag{1.3}$$

where the resonant line half-width is

$$\Delta\lambda_r = \frac{2}{3\pi} \frac{e^2}{m_a} \frac{\lambda^3}{c^2} f n_1, \tag{1.4}$$

the Stark half-width is

$$\Delta\lambda_{1/2} = 2[1 + 1.75\cdot 10^{-4} n_e^{1/4} \alpha_{III} (1 - 0.068 n_e^{1/6} T_e^{-1/2})]\cdot 10^{-16} \omega n_e, \tag{1.5}$$

and the Doppler half-width is

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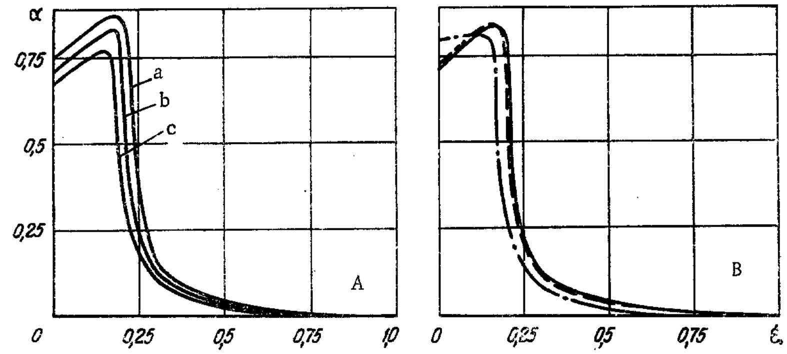

Fig. 1. Degree of ionization α in the shock layer with resonance and continuous radiation taken into account for the modifications a, b, c in A, and for the modification b with resonance and continuous radiation (the solid line) taken into account, without taking resonance radiation into account (dashes), with just resonance radiation taken into account (dash–dot curve) in B (α, ξ are dimensionless quantities).

$$\Delta\lambda_D = 7.16\cdot 10^{-7}\lambda\left(\frac{T_a}{\mu}\right)^{1/2}, \tag{1.6}$$

$$\Delta\lambda = (\lambda - \lambda_{12}). \tag{1.7}$$

The values of the constants α_m, w are presented in [5].

The volume deexcitation approximation was taken in the computation of the resonance radiation transfer to the wings of the line where the optical thickness of the compressed layer is $\tau \gtrsim 0.1$. Near the center of the line where $\tau \gtrsim 20$, it was considered that radiation is blocked. The intermediate domain of the spectrum $(20 > \tau > 0.1)$ was separated into a number of frequency bands in each of which the absorption coefficient was averaged with respect to the frequency.

As computations [3] show, a qualitative similarity between the changes in the hydrodynamic and radiation flow parameters is observed in the frontal portion of the body for different rays θ = const. As should have been expected, the strongest influence of the radiation is manifested at the zero streamline ($\theta = 0$). Since the main purpose of this paper is to clarify the influence of the various kinds of radiation on the flow around bodies, the main attention will be paid to studying the flow along the zero streamline.

The system of equations describing the gas flow in the shock layer is

$$\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{\rho}\frac{\partial \rho}{\partial r} + \frac{v}{\rho r}\frac{\partial \rho}{\partial \theta} + \frac{2u}{r} + \frac{v}{r}\operatorname{ctg}\theta = 0, \tag{1.8}$$

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{1.9}$$

$$u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta}, \tag{1.10}$$

$$\left(\rho u\frac{\partial}{\partial r} + \rho\frac{v}{r}\frac{\partial}{\partial \theta}\right)\left[\frac{V^2}{2} + \frac{5}{2}R(T_a + \alpha T_e) + \alpha R T_{i1}\right] = -\frac{\partial q}{\partial r},$$

$$\left(\rho u\frac{\partial}{\partial r} + \frac{\rho v}{r}\frac{\partial}{\partial \theta}\right)\left[\frac{5}{2}RT_e\alpha\right] = \omega_{ee} + \omega_{ie} - kT_{i1}\dot n_{ea} + \tag{1.11}$$

$$+\frac{3}{2}kT_0\dot n_{ca} + kT_e\dot n_{gr} - kT_{i2}\dot n_{ex} - en_e\vec{E}\cdot\vec{V}, \tag{1.12}$$

$$\rho u\frac{\partial \alpha}{\partial r} + \frac{\rho v}{r}\frac{\partial \alpha}{\partial \theta} = m_a(\dot n_{ca} + \dot n_{ea} - \dot n_{rad}), \tag{1.13}$$

where

$$\dot n_{rad} = \dot n_{gr} + \dot n_{ex};$$

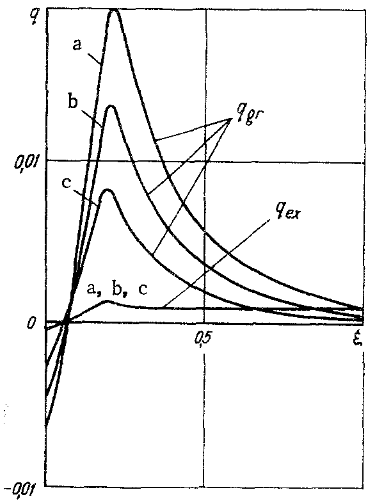

Fig. 2

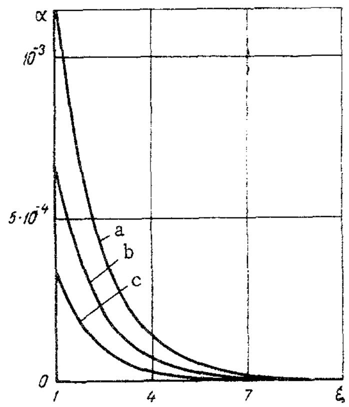

Fig. 3

Fig. 2. Profiles of dimensionless radiation energy fluxes q_{gr} and q_{ex} in the shock layer for modifications a, b, c (q, ξ are dimensionless).

Fig. 3. Change in the degree of ionization α in the heated domain for modifications a, b, c (α, ξ are dimensionless).

$$\frac{m_e}{m_a}\rho_i(\vec{V}\nabla)\vec{V} = -\nabla p_e - en_e\vec{E}; \tag{1.14}$$

$$p = \rho R(T_a + \alpha T_e). \tag{1.15}$$

The radiation transport equation, which is presented in [1, 2] in the local-one-dimensional layer approximation, is added to the system (1.8)-(1.15). This equation is used to examine resonant radiation transport in the intermediate spectrum domain and continuum radiation in the ground level. The boundary conditions for the system (1.8)-(1.15) are presented in [2, 3, 7]. The value of the degree of ionization directly behind the shock front was hence calculated from the condition of conservation of the total excitation and ionization energy of the ions and excited atoms during passage through the shock front, i.e.,

$$\alpha_- T_{i1} + \alpha_-^* T_{ex} = \alpha_+ T_{i1} + \alpha_+^* T_{ex}, \tag{1.16}$$

where α_-, α_-^* and α_+, α_+^* are, respectively, the value of α and α^* ahead of and behind the shock front. Under the conditions considered $\alpha_+ \approx \alpha_-$.

2. Formulation of the Problem for a Heated Layer

The governing process in a heated layer, which is characterized by considerably lower values of the heavy particle temperature and the electron concentration as compared with a shock layer, is atom ionization and excitation during absorption of the radiation issuing from the shock layer. In contrast to [4, 8], collision processes with the participation of the excited level were considered herein in addition to radiation absorption in the heated layer.

Using the results of [9, 10] and manipulating, we obtain

$$\dot{n}_{12} = \alpha(1-\alpha-\alpha^*)\rho^2\frac{4C_e}{m_a\sqrt{2\pi}}\sqrt{\frac{m_a}{m_e}}(RT_e)^{3/2}\left[\frac{T_{ex}}{T_e}+2\right]\exp\left(-\frac{T_{ex}}{T_e}\right)\left[1-\frac{\alpha^*}{1-\alpha-\alpha^*}\frac{g_1}{g_2}\exp\left(\frac{T_{ex}}{T_e}\right)\right], \tag{2.1}$$

$$\dot{n}_{23} = \frac{\alpha\alpha^*\rho}{(RT_e)^{3/2}}\frac{4\pi e^4}{\sqrt{2\pi m_e m_a}\, m_a^3}\left[\frac{T_e}{T_{i2}}\exp\left(-\frac{T_{i2}}{T_e}\right)-E_1\left(\frac{T_{i2}}{T_e}\right)\right]\left[1-\frac{\alpha^2}{\alpha^*}\frac{\rho}{\rho_j}\left(\frac{T_{i1}}{T_e}\right)^{3/2}\frac{g_2}{g_1}\exp\left(\frac{T_{i2}}{T_e}\right)\right]. \tag{2.2}$$

Since the extent of the heated layer is much greater than the characteristic body dimension [4], the initial system of equations in the heated layer can be written as

$$\rho u = \text{const}, \tag{2.3}$$

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dp}{dr}, \tag{2.4}$$

$$\rho u\frac{d}{dr}\left(\frac{u^2}{2} + U + \frac{p}{\rho}\right) = -\frac{dq}{dr} - \rho\alpha^* A_{21} g_{t2}\frac{h\nu_{12}}{m_a}, \tag{2.5}$$

$$\rho u\frac{d}{dr}\left(\frac{5}{2}RT_e\alpha\right) - u\frac{d}{dr}(RT_e\alpha\rho)$$

$$= \omega_{ae} + \omega_{ie} - kT_{i2}\dot{n}_{23} - kT_{ex}\dot{n}_{12} + kT_1(\dot{n}' + \dot{n}_{gr}), \tag{2.6}$$

$$\rho u\frac{d\alpha}{dr} = m_a(\dot{n}_{23} + \dot{n}_{gr} + \dot{n}'), \tag{2.7}$$

$$\rho u\frac{d\alpha^*}{dr} = m_a\left(\dot{n}_{12} - \dot{n}_{23} + \dot{n}_{ex} - \dot{n}' - \rho\alpha^*\frac{A_{21}g_{t2}}{m_a}\right), \tag{2.8}$$

$$p = \rho R(T_a + aT_e). \tag{2.9}$$

3. Method of Solution and Some Results

The following method was used to compute the flow on the zero streamline. First the flow in the whole frontal domain was considered in the formulation [3] without taking account of radiation. We hence obtained the shape $\varepsilon(\theta)$ of the shock and the dependence of the pressure on the angular coordinate $p(\theta)$. The coefficients $p_2(\xi)$ and a_2 which change slightly when radiation is taken into account, were determined from the approximate polynomials

$$\varepsilon(\theta) = \varepsilon_0 + a_2\theta^2 + a_4\theta^4, \tag{3.1}$$

$$p(\theta) = p_0 + p_2\theta^2 + p_4\theta^4 \tag{3.2}$$

Later a_2 and $p_2(\xi)$ are used to solve the problem along the zero streamline in the system (2.3)-(2.9), which becomes as $\theta \to 0$

$$\frac{du}{dr} + \frac{2}{r}V_1 + \frac{u}{\rho}\frac{d\rho}{dr} + \frac{2u}{r} = 0, \tag{3.3}$$

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dp}{dr}, \tag{3.4}$$

$$u\frac{dV_1}{dr} + \frac{V_1^2}{r} + \frac{uV_1}{r} - \frac{2\xi a_2}{r}\frac{du}{dr} = -\frac{1}{\rho r}2p_2, \tag{3.5}$$

$$\rho u\frac{d}{dr}\left[\frac{u^2}{2} + \frac{5}{2}R(T_a + \alpha T_e) + \alpha RT_{i1}\right] = -\frac{dq}{dr}, \tag{3.6}$$

$$\rho u\frac{d}{dr}\left(\frac{5}{2}RT_e\alpha\right) - u\frac{d}{dr}(RT_e\alpha\rho) = \omega_{ae} + \omega_{ie} -$$

$$-kT_{i1}\dot{n}_{ea} + \frac{3}{2}kT_0\dot{n}_{aa} + kT_e\dot{n}_{gr} - kT_{i2}\dot{n}_{ex}, \tag{3.7}$$

$$\rho u\frac{d\alpha}{dr} = m_a(\dot{n}_{aa} + \dot{n}_{ea} + \dot{n}_{rad}), \tag{3.8}$$

$$p = \rho R(T_a + \alpha T_e). \tag{3.9}$$

As in [2, 3], the method of solving the problem reduces to a double iteration. The inner iteration cycle consists in finding the position of the shock for a given radiation field distribution while the radiation field itself is determined in the outer iteration cycle. The flow without radiation is taken as the initial iteration. The outer iteration cycle is continued until both the hydrodynamic and the radiation fields agree to a given degree of accuracy in two successive iterations.

As an illustration, let us present some results of computations for the case of argon flow around a sphere with a surface emissivity $\delta = 0.5$ for a free stream Mach number $M_\infty = 30$ and a degree of ionization $\alpha_\infty = 10^{-12}$. The pressure p_∞ and the radius of the streamlined spherical bluntness L varied in such a manner that the product Lp_∞ remained constant, namely:

a $p_\infty = 0.00015$ atm, $L = 40$ cm,

b $p_\infty = 0.0015$ atm, $L = 4$ cm,

c $p_\infty = 0.015$ atm, $L = 0.4$ cm.

Since binary collisions for which the product Lp_∞ is a similarity parameter plays the governing role in the shock layer under our conditions, the values of the dimensionless standoffs ε are similar in the modifications considered. The same is true for the values of the relaxation zone widths as well as for the hydrodynamic parameters such as the pressure p, the atom T_a and electron temperature T_e, and the degree of ionization α. Therefore, the values of the optical thickness of the shock layer τ relative to continuum radiation from the ground level will also be similar in the modifications investigated.

In contrast to the preceding papers [2, 3], the absorption coefficient of continuum radiation from the ground layer is taken from [11] in the present computations.

A noticeable difference in the behavior of the hydrodynamic parameters is manifested for the modifications a, b, c only in the domain adjacent to the body surface where ternary collisions due to collision recombination become essential (see Fig. 1a).

Presented in Fig. 1b are profiles of the degree of ionization α for the modification b as a function of taking account of the diverse radiation mechanisms. It is seen that the influence of radiation transport in the resonance line is slight compared to the influence of continuum radiation transport from the ground level. Taking this latter into account results in some narrowing of the relaxation domain.

Presented in Fig. 2 are profiles of the dimensionless continuum radiation energy fluxes from the ground level q_{gr} and the resonance radiation q_{ex} for cases a, b, c. Here the values of the radiant fluxes are referred to the quantity $\rho_\infty u^2_{max}/2$, the value of the kinetic energy flux at infinity. It is seen that the energy flux on the body, transportable in that part of the line for which the optical thicknesses are $0.1 \lesssim \tau$ 20, is substantially less than the continuum radiation energy flux from the ground level. At the same time, the continuous and resonance radiation fluxes taken out in the heated layer are commensurate.

If the optical thicknesses of the shock layer were similar for continuum radiation from the ground layer in modifications a, b, c, then the optical thicknesses are proportional to the gas density for the same resonance radiation frequencies since the line width [see (1.4)-(1.5)] vary in proportion to the density. Therefore, portions of the line more remote from the center should be considered for variations with a large value of ρ_∞ (or p_∞).

Let us turn to an examination of the heated layer. Emergence of radiation in the heated layer has practically no influence on such parameters as ρ, u, T_a and substantially alters the values of α, α^* and T_e. Because of resonance radiation absorption in the heated layer, excited atoms originate whose concentration α and α^* grows at the wave front to a 10^{-5} order of magnitude. The electron concentration ahead of the front is related to absorption of continuum radiation from the ground level and is on the order of magnitude of 10^{-3}.

The continuum radiation flux from the ground level to the heated layer diminishes 100-fold in a length on the order of the body radius, while the domain of high values of T_e in the heated layer has a correspondingly similar dimension. A rise in the electron concentration α is observed in this same domain, which occurs mainly because of absorption of continuum radiation from the ground level.

The resonance radiation domain we considered was divided into six frequency bands. It should be noted that the value of the total flux also depends on the method of partitioning the line in the frequency bands. The resonance radiation path length in the heated layer is considerably greater than for the continuum, where the maximum path length (for the most remote frequencies from the center of the line) is a quantity on the order of hundreds of body radii. The domain in which a rise in concentration of the excited atoms α^* occurs is related to the depth of resonance radiation penetration into the heated layer.

The optical thicknesses for continuum radiation from an excited level are much less than the optical thicknesses for all other kinds of radiation considered. Hence, taking account of absorption of continuum quanta from the excited level in the heated layer influences the gasdynamic parameter profiles slightly. The electron temperature varies most strongly. It approximately doubles in the far heated layer as compared to the value at infinity.

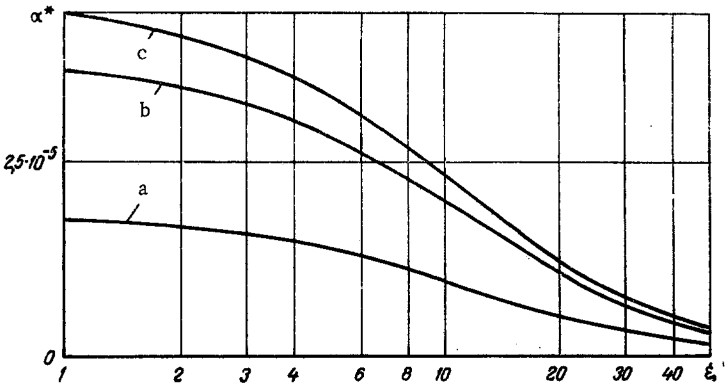

Fig. 4. Change in excited atom concentration α^* in the heated domain for modifications a, b, c (α^*, ξ are dimensionless).

Taking account of transmission of the internal energy of the excited atoms to the electron gas because of impacts of the second kind turns out to be insignificant under the conditions we considered.

NOTATION

$\vec{r}$, radius vector; ξ, dimensionless radius vector; θ, polar angle, $\vec{V}$, u, v, velocity vector and its components along the radius vector and along its normal; $V_1 = (\partial v/\partial\theta)_{\theta=0}$; p, total pressure; p_e, electron gas pressure; ρ, gas density; ρ_i, ion gas density; α, degree of gas ionization; α^*, relative excited atom concentration; T_a, T_e, atom–ion and electron gas temperature; T_0, temperature of electrons being formed during atom–atom collisions; T_1 temperature of electrons being formed during radiation absorption in the heated layer; m_a, m_e, atom and electron mass; e, electron charge; R, specific gas constant; k, Boltzmann constant; h, Planck's constant; T_{i_1}, T_{i_2}, ionization temperature from the ground and excited levels; ν, frequency; λ, radiation wavelength; ν_{12}, λ_{12}, frequency and wavelength at the center of the resonance line; $\dot{n}_{ea}$, $\dot{n}_{aa}$, ionization rate by electron–atom and atom–atom impact; $\bar{n}_{12}$, $\bar{n}_{23}$, rate of excitation of the ground and ionization of the excited levels by electron impact; $\dot{n}_{gr}$, $\dot{n}'$, rate of photoionization by continuum radiation from the ground and excited levels; $\dot{n}_{ex}$, rate of excitation in resonance radiation absorption; ω_{ie}, ω_{ae}, rate of energy exchange in ion–electron and atom–electron collisions; $\vec{E}$, polarization field intensity; U, internal energy of unit mass of gas; q_{gr}, q_{ex}, continuum radiation energy fluxes from the ground and resonance levels; $q = q_{gr} + q_{ex}$; M_∞ unperturbed stream Mach number; n_e, electron concentration; n_k, atom concentration in the ground state; L, radius of the spherical bluntness; g_1, g_2, statistical weights of the atom ground and excited states; $\varkappa(\nu)$, absorption coefficient in the continuum from the ground level; k_0, resonance absorption coefficient at the center of the line; f, oscillator strength; $C_e, \rho_j, \alpha_{III}, w, A_{21}$, constants; g_{t2}, Gaunt factor.

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